Sonuç: a ve b; $p \times 1$ sabitlerden oluşan vektörler ise

$$Cov(a'y; b'y) = a' \cdot \Sigma \cdot b \text{ olur.}$$

$z = A \cdot y$, $w = B \cdot y$ olsun. $A_{k \times p}$, $B_{m \times p}$ sabitler matrisi, $y_{p \times 1}$ rastgele ~~ile~~ Σ kovaryans matrisi ile vektör,

1: $Cov(z) = Cov(A \cdot y) = A \cdot \Sigma \cdot A'$

2: $Cov(z, w) = cov(A \cdot y, B \cdot y) = A \cdot \Sigma \cdot B'$.

Örnek: $y = (y_1 \ \ y_2 \ \ y_3)'$ rastgele vektör olsun.

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix}$$

olmak üzere, a.) $z = 2y_1 - 3y_2 + y_3$ için

$E(z)$, $V(z) = ?$

b.) $z_1 = y_1 + y_2 + y_3$, $z_2 = 3y_1 + y_2 - 2y_3$ için

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ olsun. $E(z)$, $Cov(z) = ?$

Çözüm: a.) $z = 2y_1 - 3y_2 + y_3$ t.d. için

$$E(z) = E(2y_1 - 3y_2 + y_3) = 2 \cdot E(y_1) - 3 \cdot E(y_2) + E(y_3)$$
$$= 2 \cdot 1 - 3 \cdot (-1) + 3$$
$$= 2 + 3 + 3 = 8$$

$$V(Z) = V(a_1y_1 + a_2y_2 + a_3y_3) = a'.\Sigma.a$$

$$= \begin{bmatrix} 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = 2 \,//$$

(b) $Z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ iki bileşenli

$$\Rightarrow E(z) = E\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 - 1 + 3 \\ 3 - 1 - 6 \end{bmatrix} = \begin{bmatrix} E(y_1) + E(y_2) + E(y_3) \\ 3.E(y_1) + E(y_2) - 2.E(y_3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -4 \end{bmatrix} \,//$$

$$Cov(Z) = Cov\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{bmatrix} \sigma_{z_1}^2 & \sigma_{z_1z_2} \\ \sigma_{z_2z_1} & \sigma_{z_2}^2 \end{bmatrix}$$

burada; $\sigma_{z_1}^2 = V(z_1) = V(y_1 + y_2 + y_3)$

$$= a'.\Sigma.a$$

$$= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 13 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 21 \,//$$

$$\sigma_{z_2}^2 = V(z_2) = V(3y_1 + y_2 - 2y_3)$$

$$= 3^2.V(y_1) + 1^2.V(y_2) + (-2)^2.V(y_3) + 2.3.1.\sigma_{12}$$

$$+ 2.3.(-2).\sigma_{13} + 2.1.(-2).\sigma_{23}$$

$$= 9.1 + 2 + 4.10 + 6.1 - 12.0 - 4.3$$

$$= 9 + 2 + 40 + 6 - 12 = 45 \,//$$

$$\sigma_{z_1 z_2} = cov(z_1, z_2) = cov(A.y\,;\,B.y)$$

$$= A . \Sigma . B'$$

$$= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix} . \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 13 \end{bmatrix} . \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = 6 + 6 - 26 = -14$$

Böylece varyans-kovaryans matrisi;

$$Cov(z) = \begin{bmatrix} \sigma_{z_1}^2 & \sigma_{z_1 z_2} \\ \sigma_{z_2 z_1} & \sigma_{z_2}^2 \end{bmatrix} = \begin{bmatrix} 21 & -14 \\ -14 & 45 \end{bmatrix}$$

şeklinde elde edilir.

$$z \sim N(\mu_z, \Sigma_z)$$

$$= \left[\begin{pmatrix} 3 \\ -4 \end{pmatrix}, \begin{pmatrix} 21 & -14 \\ -14 & 45 \end{pmatrix} \right]$$